Bobby Learns His Numerals

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THE following scenes from the history of number were written for presentation by students of junior, and of first year senior, high school age. They do not pretend to give a complete account of the development of the number system. They were given in a twenty minute assembly period, and, for dramatic purposes, we chose to represent the characters as children of the age of the actors and to make the action swift rather than the treatment of the subject complete.

Each scene was accompanied by a chart showing the numerals of the period. The charts were made in collaboration with the Art Department. They were of heavy white cardboard with numerals six inches high and $\frac{1}{4}$ to $\frac{1}{2}$ inches wide and were visible in all parts of the auditorium. They were placed on an easel by an attendant in costume who knew her cues and did the work swiftly and unobtrusively. A lantern and screen could be used instead of the charts. The use of charts has two advantages which we did not foresee. The charts were hung on exhibition in the hall after the presentation of the scenes, and the students enjoyed studying them at leisure. Also, they are surprisingly beautiful. The Egyptian and Chaldaean symbols have, perhaps, the most intrinsic beauty, but it is gratifying to see that a work of art can be made from our prosaic digits. A list of the charts follows this article.

The class elected a committee of students to study the proper costuming. We did not wish to go to any expense, so the costumes were assembled from material in the school property-room and from articles lent by our friends. Costumes are not necessary, but they add interest and color and make an artistic whole.

Some of the properties were made in the crafts shop. Among them were a painted headdress and collar for the Egyptian boy, the clay tablet of the Chaldaean, an abacus, and the bamboo book of Lilivati.

The entire project made an interesting correlation of mathematics with history, art, and crafts.

CHARACTERS

BOBBY, *an American boy.*
BOBBY'S MOTHER.
AHMES, *an Egyptian boy of 2000 B.C.*
ABR-AM, a *Chaldaean boy of about 2000B.C.*
THEON, *a Greek of the classical period,*
GAIUS, LUCIUS } *two Roman boys of the Augustan age.*
LILAVATI, *a Hindu girl.*
AL-KHOWAZMI, OMAR-KHAYAM } *two Arabian boys.*
LEONARDO OF PISA, *an Italian boy.*

With the exception of Bobby's mother, the characters are represented as children about twelve years old. Lucius is younger.

The curtain rises showing a boy about twelve years old, dressed in pajamas and dressing-gown, working over his arithmetic. His table is strewn with books and papers.

VOICE FROM OFF STAGE. Sleepy-head, sle-e-py-head! Time to put away your books!

BOBBY. Oh, mummie, just let me finish this example! I've got it almost done!

VOICE. Well, ten minutes more. But look out or you'll dream about figures.

BOBBY. I wish I knew who invented all this old stuff anyway. Wish I'd lived a thousand years ago before there were any fractions to plague me!

(*Bobby works on, growing drowsier and drowsier. His head drops on his forearm and he is fast asleep.*

The curtain falls for a few seconds. When it rises, the screen on which the numerals are to be projected, or an easel with charts and attendant, is on the stage.

A boy in the Egyptian costume of 2000 B.C. enters.)

BOBBY. Who are you?

AHMES. I'm Ahmes. I lived in Egypt more than three thousand years ago. I heard you say that fractions bother you, so I came to sympathize with you.

BOBBY. What, did you have fractions?

AHMES. Oh yes, and problems and equations. Hard ones, too.

BOBBY. What were they like?

AHMES. Our earliest numerals were on stone monuments and were like these hieroglyphics on a royal tomb which show the number of the king's flocks. (*Chart I. Points and reads the numbers*).

Each of these figures tells a story. (*Chart II*) The numbers up to ten were merely scratches, two straight lines for two, six for six (*points to them as he names them*) One hundred is a coil of rope such as our surveyors used in measuring the fields; a thousand is a lotus flower, we had so many of them. Ten-thousand a man's hand, pointing in amazement. You'd never guess one-hundred thousand! A tadpole, because the mud of the Nile swarmed with them after each flood!

BOBBY. Yes, I remember about the plague of frogs. But how did you write a large number?

AHMES. This is the number of our king's captives. (Points it out on Chart III and reads it.)

BOBBY. What about those fractions?

AHMES. They were a nuisance. We had to change them all to a common numerator, one. (*Points to* $\frac{1}{4}$) This is one-fourth.

BOBBY. We change them to a common denominator.

AHMES. They have always been hard. Our teachers made tables to help in reducing them, but they were still difficult.

BOBBY. You said you had to solve problems?

AHMES. Yes, practical ones, such as the manager of a plantation needs in distributing bread to his slaves and mechanics. We were great agriculturists. We built granaries to store our wheat and millet.

BOBBY. Like our grain elevators?

AHMES. Somewhat; and we had to figure their cubical contents.

BOBBY. Whew! Did you have formulas and equations to help?

AHMES. Here's one. (*Chart IV. Reads and points*) Heap, that's your x, its whole, its seventh, it makes 19. The legs walking forward mean addition.

BOBBY. I've seen that in an ancient papyrus. My teacher says it is the oldest written equation. We should write $x+x/7=19$. (*writes*)

AHMES. Here is a problem I have always liked. It is in that same papyrus. (*Chart V*) See the numbers mount. Houses, 7; Cats, 49; Mice, 343; Ears of Corn, 2401; Grains of Corn, 16807.

BOBBY. Why, that's like our riddle,

Kits, cats, sacks and wives,

How many were going to St. Ives?

Ahmes exit. A black-haired, curly-headed boy rushes in, holding out a clay tablet which he displays, saying:

ABR-AM. I'm Abr-am from Ur of the Chaldees over between the Tigris and Euphrates rivers. See what big numbers I had to learn!

BOBBY. Tell about them.

ABR-AM. My people were traders and lawmakers. In their work they used numbers like these wedge shaped marks pressed in soft clay bricks with a stylus and baked. (*Chart VI*) The vertical wedges are units; the horizontal, tens. This is fourteen.

In ordinary business we counted by tens as you do, but I went to the temple school at Bel-Nippur, where we counted by sixties. My old tablets are still in the ruins there. Here is one. (*Chart VII*) This line says "one 60 plus seven 10s equals two 60's plus one 10."

BOBBY. Why did the priests use sixty as a unit?

ABR-AM. Because they were astronomers. Our ancestors were wandering tribes. They came to love the stars which guide them across the desert, and to worship them.

BOBBY. But what has sixty to do with that?

ABR-AM. Our astronomer-priests divided the circle into three hundred and sixty equal parts, perhaps because the year has three hundred and sixty-five days. We don't know, it was all so long ago. And they divided the great circle of the heavens into twelve regions, (*Chart VIII*) the signs of the Zodiac. You remember 60 goes six times into 360, even as the radius goes six times around the circle. (*Points to figures of the inscribed hexagon and the six-pointed star*)

BOBBY. Is that how we get our circular measure of 360 degrees and 90 degrees in a right angle?

ABR-AM. Yes, and your signs of the Zodiac, and your days of twenty-four hours of sixty minutes of sixty seconds each. They all come from our wise men who studied the stars. (*Exit*)

BOBBY. I wonder who comes next.

Enters a Greek boy, Theon.

THEON. I should come, but I never learned to do much reckoning. We Athenians left most of the business of calculating sums to our slaves. Oh, we did a little addition on counting frames, but we never worked out a really good system of writing numbers (*Chart IX*). We first used the initial letters; pi stood for pente, five, mu for myriads. Later we used the letters of the alphabet in order with a system of primes. (*Reads from the chart*)

BOBBY. Then you never studied arithmetic?

THEON. But yes, the properties of numbers! Odd numbers, even numbers, prime numbers, numbers built up in triangles and squares. (*Chart X*) Our great Pythagoras thought the universe was built on number. He offered a sacrifice of a hundred oxen when one of his brotherhood discovered the irrational number, the diagonal of the square, root-two, (*Points to chart*)

BOBBY. Sounds sort of looney to a plain American boy. Tell me about that counting frame.

THEON. There are Gaius and Lucius on their way to school. They will show you. They like practical things.

Enter two Roman boys, Gaius, a lad of fourteen, carries an abacus, a bag of counters, stylus and tablets. Lucius, a little boy, is speaking to him.

LUCIUS. Oh Gaius, see me do my finger counting. (*Holds up his hand, makes the counting signs, and counts*) unus, duo, . . . decem.

GAIUS. Good, little one.

LUCIUS. But I can't do my sums.

GAIUS. Show me one.

LUCIUS. (*Pointing to Chart XI and reading*) 278 et 64. These are too large for finger counting.

GAIUS. On this abacus put 8 and 4 counters on the first line, 7 and 6 on the next, 2 on the third.

LUCIUS. Yes.

GAIUS. Take ten off the first and put one in place of them on the second column. Take . . .

BOBBY, interrupting. So that's why we say we "carry one!"

GAIUS, continuing. Take ten off the second and put one on the third. How many have you?

LUCIUS. 342.

GAIUS. Optime!

BOBBY. That is clever. But none of you can multiply and divide with your numerals. And you have so many of them! We use only nine little figures and a zero, yet we can write huge numbers with them and make long computations. (*Chart XII*). Where did we get these queer symbols (*points to a three and an eight*) with which we do these long examples compactly and quickly?

Enter a girl of twelve years in Hindu costume.

LILAVATI. I think I can answer that question.

BOBBY. You, a girl!

LILAVATI. Yes, my father, Bhaskara, was a scholar and a poet. He wrote all his knowledge in a book of verse for me. There is much about numbers in it, and algebra.

BOBBY. I didn't suppose girls had to study such things in your time.

LILAVATI. They didn't, but you see my father learned from the stars the day and hour propitious for my marriage. The hour cup was floating on the stream and we were watching for the water to rise to mark the fortunate moment, when a pearl from my marriage headdress fell and closed the opening. The proper time passed unnoticed and I was destined never to marry. To console me, my father wrote a book in my name, Lilavati, the beautiful, that my name might be known to latest times.

BOBBY. I call that hard luck. But about the numbers?

LILAVATI. On the walls of a cave in India are strange marks, probably made by traders who stopped there for shelter over night. (*Chart XIII*) Many people think these are the earliest forms of the numbers you use.

BOBBY. Did all your writers use these figures?

LILAVATI. No, our mathematicians wrote in Sanskrit (*points to Sanskrit on chart*), and their problems were in poetry, often very fanciful. The numbers were written out as words. Sometimes they were named after colors.

BOBBY. What kind of problems?

LILAVATI. About pipes filling cisterns, and the number of bees in a swarm. (*Reads one from her book*) The square root of half the number of bees in a swarm, and also 8/9 of the whole, alighted on the jasmines; and a queen bee buzzed responsive to the hum of a male inclosed in a lotus-flower, into which he had been allured at night by its sweet odor. Tell me, O beautiful damsel, the number of the bees.

BOBBY. And they could solve problems like that with words!

LILAVATI. And could complete the square of a quadratic. They showed the world how to do that.

But long before my day these nine digits which common traders employed had come into use and their knowledge had been carried across the great mountains by strangers from the west. There are two of them now.

Two Arabian lads enter, dressed in costumes of the period of the "Arabian Nights."

AL-KHOWARIZMI. Our forefathers were fierce Arabian tribesmen. By the sword they carried our religion through the dread Khyber Pass and to the Pillars of Hercules. Our caliph, Haroun-al-Raschid, brought to his court at Bagdad wise men from India and sages from Alexandria. We studied the Euclid of the Greeks and the algebra of India. By the way, you know, of course, that algebra is an Arabic word?

BOBBY. Meaning?

OMAR-KHAYAM. The "crossing over," because we learned to simplify the process of solving equations by transposing terms across the equality sign.

BOBBY. Gee! And you used the nine Hindu numerals?

OMAR. Yes. Many people today call them the Arabic numerals. (*They turn to go*)

BOBBY. Wait. Won't you tell me your names?

AL-K. This is Omar-Khayam. He was a good poet, but a better algebraist, and I am Al-Khowarizmi.

Leonardo of Pisa, commonly called Fibonnacci, an Italian boy of the twelfth century, enters. He carries a large leather-bound book.

LEONARDO. You know the rest of the story, don't you?

BOBBY. Do I?

LEONARDO. About the Crusades?

BOBBY (*puzzled*). Yes.

LEONARDO. And the Crusaders returning with gifts, silks and gems, perfumes and spices for wives and sweethearts, so that commerce developed between the Orient and Europe?

BOBBY. Oh yes, and the growth of the rich city republics of Italy. We had that in history.

LEONARDO. And trade routes down the Rhine and the Hanseatic League?

BOBBY. Of course. I begin to see.

LEONARDO. Well, that is the route the Arabic numerals traveled. It was a good thing, too, for the increased needs of commerce and banking required a better number system than the Roman and other ancient methods of notation.

BOBBY. What did you have to do about its introduction?

LEONARDO. My father was an official of Pisa. You would call him a consul, looking after the interests of our merchants in a seaport of North Africa. I used to play around the counting house and the harbor. I talked with the sailors. I saw the caravans come in with bales of merchandise and picked up some of the language of the swarthy Arab traders. They had a quick way of reckoning. When I grew up and had returned to Italy, I wrote about their numbers in my book on arithmetic.

BOBBY. Then you were the first European to use them?

LEONARDO. No, not exactly. One of the popes, Sylvester II, as a young man learned about them in Spain where there were Saracen universities. But he used them only to number the columns on his abacus. He did not reckon with them.

BOBBY. Weren't your countrymen awfully glad to learn about the new reckoning?

LEONARDO. No, they were very slow about taking it up. Some universities even passed laws forbidding the use of the Arabic numerals. The methods were long and clumsy at first.

(*Shows Chart XV and with pointer explains process of Gelosia Multiplication*)

(*Chart XVI*) The arrangement of this longer example shows why the method was called the Gelosia or "grating" method.

(*Chart XVII*) In division many figures used in the process were erased or scratched out.

(*Chart XVIII*). The work in long division was often arranged in a form which suggested a galley, a kind of ship propelled by oars. This example, taken from the first printed arithmetic, illustrates this so-called galley method of division. Notice that the boy who worked the problem, decorated it as you sometimes ornament your papers.

BOBBY. My! I guess arithmetic always has been hard.

LEONARDO. Just as hard for the human race as for you. But we have learned it.

Exit Leonardo. Curtain, during which time charts or screen are removed.

Curtain rises showing Bobby at his desk, sleeping with head on his arm. Enters Mother. She shakes him gently to arouse him and picks up the scattered books and papers,

MOTHER. Bobby, Bobby, wake up, dear.

BOBBY (*triumphantly*). Oh Mummie, I got that last example!

Curtain.

CHARTS

I. Inscriptions from a royal tomb in Egypt with hieroglyphics, showing numbers of the king's flocks and herds. From Karpinski's History of Arithmetic. Rand McNally Co.

II. Egyptian hieroglyphs for 1, 2, 4, 10, 100, 1,000, 10,000, 100,000, ½.

III. 212,346 in hieroglyphics.

IV. Two equations in hieroglyphics. a) Problem 33. Plate 55. b) Problem 27, Plate 47 from Rhind Papyrus, ed. by Chace, Bull, Manning.

V. Problem 79, Plate X, from Rhind Papyrus (The geometric progression).

VI. Chaldaean cuneiform numerals. Tablet from Bel-Nippur showing decimal and sexagesimal equivalents from Hilprecht, reproduced in a German history of mathematics. Sigmund Gunther, Sammlung Schubert XVIII, Leipzig.

VII. The Zodiac, signs and symbols arranged in a circle, in black and gold. The inscribed regular hexagon. The six pointed star.

VIII. Greek numerals. Capital initial letters. Later primed letters used in numbers, Triangular numbers of the series 1, 3, 6. 10 in geometric form. Square and its diagonal to illustrate root-2.

IX. Roman numerals for 1, 2, 4, 5, 10, 50, 100, 1,000. 278 plus 64 in Roman numerals.

X. The nine Arabic numerals and zero. A computation in these numerals.

XI. Copy, in color, of inscriptions in cave at Nana Ghat. Detail from these inscriptions. Sanskrit numerals 1 . . . 4. From D. E. Smith, History of Mathematics. Vol. 1.

XII. Development of the figure 8. From Fink, Brief History of Mathematics. Tr. W. W. Beman and D. E. Smith, Open Court.

XIII. A short example showing Gelosia multiplication.

XIV. A longer example showing Gelosia multiplication.

XV. An example showing scratch method of division. XIV, XV, XVI are from the Treviso Arithmetic, reproduced in D. E. Smith, History of Mathematics.